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Morphing UAV Pareto Curve Shift for Enhanced Performance

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Research in unmanned aerial vehicles (UAVs) has grown in interest over the past couple decades. Historically, UAVs were designed to maximize endurance and range, but demands for UAV designs have changed in recent years. In addition to the traditional demands for endurance and range, today customer demands include maneuverability. Therefore, UAVs are being designed to morph, to change their geometrical shape during flight, for enhanced maneuvering capability. In this investigation the morphing UAV concept under study is referred to as the buckle wing. The design of the buckle-wing airfoil geometries is posed as a multilevel, multiobjective optimization problem. This buckle-wing design problem includes two competing objectives of maneuverability and long range/endurance. Multiobjective problems have many optimal solutions each depicting a different compromise scenario. Each optimal solution is a Pareto point, and the set of all these points represents the Pareto curve. This is a powerful means of showing the global picture of the solution field. The goal of this paper is to explore and compare the Pareto curves of the buckle-wing UAV to that of a conventional non-morphing UAV. In order to make this performance comparison, Compromise Programming is used as the optimizing method, and the Vortex-Panel Method is used in calculating the aerodynamics. The buckle-wing UAV's enhanced capabilities are demonstrated both quantitatively and graphically.

Nomenclature

α	Angle of attack
cd	Drag coefficient
cl	Lift coefficient
Ma	Mach Number
ReL	Reynolds Number

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I. Introduction

A. Morphing UAV Background

Research in unmanned aerial vehicles (UAVs) has grown over the past couple of decades. Interest in UAVs is growing in interest because of the dangerous missions they can accomplish without the risk of human life. In particular, UAVs are of great interest to the U.S. Air Force, and funding from the Air Force makes the research of this project possible. UAV missions include video and IR surveillance, relay communication links, and detection of biological, chemical, or nuclear materials. These missions are autonomously controlled or piloted by remote control.

Demands for UAVs have changed in recent years. Historically, UAVs were designed to maximize endurance and range. In addition to customer needs for endurance and range, today's customer also demands maneuverability. This is especially important if the UAV encounters any resistance during the mission. A typical mission for the newer multi-role UAV includes takeoff, cruising to the desired location as efficiently as possible, maneuvering to escape problematic encounters, cruising back, and landing. During takeoff, evasive maneuvers, and landing, high lift is required and less emphasis is put on vehicle drag. For cruising however, maximizing range and endurance are desired making the lift-to-drag ratio important. The capability of obtaining maneuverability and obtaining endurance and range in the same design presents a problem because of the inherent tradeoff between the two.

Whereas the conventional UAV maximizes endurance and range, the UAV considered in this investigation is being designed to morph for an enhanced capability for maneuverability. The UAV is designed to morph; that is, it is designed to change geometrical shape during flight. This project investigates the unique buckle-wing UAV design project in the Aerospace and Mechanical Engineering Department at Notre Dame.[2] Figure 1 depicts a simulated buckle wing in flight.

Programs for morphing aircraft research have been emerging in recent years. DARPA announced its Morphing Aircraft program in *Aviation Week and Space Technology* in April 2002.[11] NASA also has a large Aircraft Morphing program.[13] A team from Purdue University, supported by NASA, recently generated 35 mission concepts for morphing aircraft.[5] Furthermore, the Purdue team presented a remarkable concept looking at morphing as an independent variable.[6] The team's approach uses aircraft performance, size, and weight as functions of morphing to size an aircraft. Another interesting area of morphing aircraft research examines the flying mechanics of birds and apply them as morphing principles on larger air vehicles.[1] On the system level, they examine the effects of variable lift-to-drag ratio and specific fuel consumption of the vehicle in cruising flight.

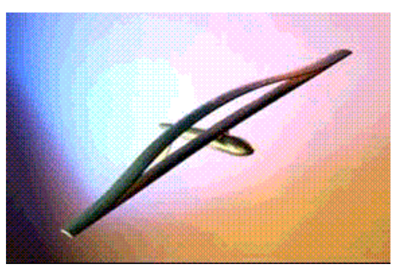

Figure 1. Buckle-Wing UAV

The morphing concept of the buckle-wing UAV has the ability to change its wing configuration from a single wing into two wings joined at each end. The research focus of for this project consists of designing the buckle-wing UAV's airfoil cross-sections. The probrem is posed as a single airfoil which splits into two airfoils when morphed. The first configuration is called fused. It consists of a single airfoil, and its objective is to optimize endurance and range via lift-to-drag cl/cd. The second configuration is called split, or the buckle wing, because it consists of that same single fused airfoil now only split into two airfoils. This configuration is designed to optimize maneuverability via lift cl.

The buckle wing has many advantages over a conventional UAV design, in that the tradeoff between maneuvering, and endurance and range can be somewhat decoupled. The buckle-wing concept allows greater performance of each capability as compared to any single design with competing objectives. However, it also presents new design challenges. These two competing objectives create a multiobjective problem for the two wing configurations.

The design problem is formulated as a multiobjective optimization problem consisting of two competing objectives. Maximizing one objective occurs at the expense of the other, or increasing one objective results in a decrease in the other. In such cases the designers must make a compromise in order to obtain satisfactory objective values. A multiobjective optimization problem has many optimal solutions each depicting a different compromise scenario. Each solution presents a point, and the set of all these points represents the Pareto curve. The Pareto curve is a powerful graphical tool to show the global tradeoff picture. There are a number of procedures proposed in research journals and textbooks for generating a compromise point. The Pareto curves of the buckle-wing UAV and a conventional UAV are generated from the method of Compromise Programming[9] in this investigation.

The goal of this paper is to explore and compare the Pareto curve of the buckle-wing UAV with that of the conventional UAV in order to quantitatively demonstrate the buckle wing's enhanced capabilities. The superiority of the buckle wing will be immediately identifiable.

B. Compromise Programming Background

Goal Programming (GP) is the background for the method used in this research. Based on the works of Steuer,[8] the variation of GP used in this investigation is described in Tappeta[9] and otherwise known as Compromise Programming.

Compromise Programming (CP) is a powerful method of solving multiobjective optimization problems. CP has the advantage of generating efficient solutions even when the Pareto curve is nonconvex. It is based on minimization of the distance between a feasible point and an unattainable point called the ideal point. The feasible point $\overline{f}_i$ is the feasible optima the designer wishes to obtain. It is actually a vector quantity and also known as the aspiration point. The unattainable, ideal point f_i^u is also called the utopian point. This too is a vector. As its name suggests it contains the unattainable, ideal solution of each objective function.

The CP optimization problem is described as

$$\begin{array}{ll} Minimize & z + \alpha \sum_{i=1}^{m} w_i f_i(x) \\ Subject\ to: & z \geq w_i(f_i(x) - f_i^u),\ i = 1, ..., m \\ & g_j(x) \geq 0,\ j = 1, ..., p \\ & h_k(x) = 0,\ k = 1, ..., q \\ & x_l^{ub} \geq x_l \geq x_l^{lb},\ l = 1, ..., n. \end{array} \tag{1}$$

In this standard form of the CP problem, the objective functions $F(x) = (f_1(x), f_2(x), ..., f_m(x))$ must be minimized. The weights w_i are determined by $w_i = 1/(\overline{f}_i - f_i^u)$, and are determined before solving the CP problem. α is an optimization parameter set to a sufficiently small positive number such as 10^{-6}.

The solution of the CP problem produces a single point. This solution is a Pareto point if it meets the Pareto optimality condition. A vector of x^* is Pareto optimal if there exists no feasible vector x which would decrease some objective function without causing a simultaneous increase in at least one objective function.

II. Research

A. Compromise Programming Implementation

Because the UAV airfoil design problem seeks to maximize the objective functions, the CP problem formulation for minimization must be changed. Several changes must take place to change the formulation into a maximizing one. First, maximizing an objective function is analogous to minimizing the objective function's negative value. Mathematically, this is stated as

$$Maximize\, F(x) \;=\; Minimize\; -F(x).$$

Additionally, the utopian point f_i^u is no longer the ideal minimal point, but it is a maximum ideal point greater than any value the designer could hope to achieve. Therefore, the weights have to be changed as well. The weights become $w_i = 1/(f_i^u - \overline{f}_i)$.

$F(x) = f_1(x), f_2(x)$. f_1 represents the first objective for endurance/range cl/cd, and f_2 represents the second objective for maneuverability cl. Since this is a maximizing problem, the CP formulation is

$$\begin{array}{ll} Minimize & z - \alpha \sum_{i=1}^{m} w_i f_i(x) \\ Subject\ to: & z \geq w_i(f_i^u(x) - f_i),\ i = 1, ..., m \\ & g_j(x) \geq 0,\ j = 1, ..., p \\ & h_k(x) = 0,\ k = 1, ..., q \\ & x_l^{ub} \geq x_l \geq x_l^{lb},\ l = 1, ..., n \end{array} \tag{2}$$

Choosing a specific set of weights defines the optimization problem. The problem then can be solved for a solution according to those weights or aspiration values. If an optimal solution is found, and its weights were chosen properly, the solution represents a Pareto point. All these points, corresponding to the different weights used in obtaining the optimal solutions, depict the Pareto curve.

Aspiration values should be chosen that are obtainable. The idea is to choose values which are just beyond the previous solution in order to find the true Pareto point. Choosing aspiration values which are too low will generate a solution, but generally this will not be a Pareto point. The aspiration targets are effectively used as weights in CP multiobjective optimization. Aspiration values were chosen such that the Pareto curve could be generated in the shortest number of trials.

In the conventional UAV design problem generally cl values around 2.0 could be achieved depending upon the airfoils selected as basis functions (cf. section D). For the buckle-wing problem achievable cl values are closer to 3.0. In both cases cl/cd values could achieve values of around 150.

The process is very intuitive and important. In order to achieve high values for one of the objectives, the corresponding aspiration values would need to be set high at the expensive of the other aspiration value. However, if the other aspiration value is too low, then the problem may converge without generating a Pareto point. Because computational time is the limiting variable in performing this research, it is really important to set the aspiration values wisely in order to minimize the number of times necessary to run the code to generate the Pareto curve.

B. Conventional UAV Airfoil Design Study

The conventional UAV airfoil design study is a single-level, multiobjective problem implemented in two variations. The first variation makes use of eight design variables representing the weights of the eight basis functions, and the second variation makes use of only three basis functions. Each also makes use of the pseudo design variable z. Each optimization problem also loads the data for the spline of each basis function. In the case consisting of eight design variables, the objective is to determine the three most dominant basis functions. Then, the second variation is run for the comparison study of the conventional and buckle-wing Pareto curves. Both variations minimize the objective function

$$F = z - \alpha(w_1 f_1 + w_2 f_2), \tag{3}$$

where z is the pseudo design variable, f_1=cl/cd, and f_2=cl. Unlike the buckle-wing study, the conventional study involves only a single wing. Hence, cl/cd and cl are the values for this single wing. The lower bounds of the design variables are 0. The weights of the basis functions are not allowed to be negative to ensure a valid design. The upper bounds of the design variables are 1.5. This number is chosen to ensure that the airfoils do not become too thick. These airfoils are appropriate selections for aircrafts such as the UAV, and allowing them to become much thicker would not be suitable for design. Likewise, another constraint guarantees that the sum of the design variables does not exceed 1.5 for the same reason. The other constraints are the two CP constraints. These constraints tend to drive the optimization process through the design variable z, which is minimized as part of the objective. The optimizer calls the aerodynamic analysis code (i.e., the panel method).

The aerodynamic analysis code calculates lift and drag. It is a modified version of Pablo,[12] which was developed in 1999 in Stockholm, Sweden. The aerodynamic analysis code uses a two-dimensional panel method. It consists of a two-step process. First, lift is calculated. Then, using this result an estimate for drag is found. This process takes approximately one-half second for a single analysis. A two-dimensional vortex panel method[3] calculates drag sectional lift and the pressure distribution. A boundary-layer model calculates the drag employing Thwaites' equations for the laminar region and Head's equations for turbulent region.[4] The drag coefficient is then computed using the Squire-Young formula.[7]

C. Buckle-Wing Problem

The buckle-wing problem is a multilevel, multiobjective optimization problem. The problem consists of three design variables and a fourth pseudo design variable. The upper-level objective function is to minimize

$$F = z - \alpha(w_1 f_1 + w_2 f_2),$$

where z is the pseudo design variable, f_1 is cl/cd_{fused} or cl/cd in the fused configuration, and f_2 is cl_{split} or the cl in the buckle-wing (i.e., split) configuration. The same constraints from the conventional problem are also imposed upon this problem. The upper-level optimizer calls upon the aerodynamic analysis code to calculate the cl/cd_{fused}, and cl_{fused}, and the optimizer calls upon the lower-level optimizer to optimize a cut and calculate the cl/cd_{split} and cl_{split}.

The lower-level optimizer consists of three design variables representing the control points. The control points lie upon a cubic spline curve used to cut the fused airfoil. This suboptimization problem additionally contains three fixed points, which also lie upon the cubic spline curve. They are located at the leading edge, the trailing edge, and one near the trailing edge to ensure that the split airfoil has a sharp trailing edge.

The suboptimizer produces a cut based on the objective and constraints. The objective is to maximize the buckle wing's split configuration cl. The lower bounds are set to ensure that the lower split airfoil contains at least 30% of the original airfoil. The upper bounds ensure that the upper split airfoil also contains at least 30% of the original airfoil. Another constraint is added to guarantee that the buckle wing's cl/cd is greater than 60. The suboptimizer calls upon a subroutine to make the cut and calls upon the aerodynamic analysis code to calculate the buckle wing's cl and cl/cd. This suboptimization process was developed in Gano.[2]

This optimization problem has many parameters. It is important to keep in mind that the goal of this project is to compare the Pareto curves of the buckle-wing UAV to that of a conventional UAV. Therefore, varying parameters is superfluous to achieving the main goal. Consequently, the Reynolds number ReL, angle of attack α, and Mach numbers Ma were kept constant and given real values which the UAV would see in its mission. These values are shown in Table 1. These values correspond to realistic ones experienced in UAV applications.

Table 1. Optimization Parameters and Their Values

Parameter	Value
ReL	5e5
α	2
Ma	0.3

D. Basis Functions

Basis functions are used to describe the shape of the airfoil. This is a technique given by Vanderplaats.[10] An airfoil is typically represented by a cubic spline and defined by over 100 data points. Because these points are so numerous, making each of these points a design variable is impractical for computational fluid dynamics (CFD) or other computational analyses such as panel codes. Therefore, basis functions are used, where each basis function represents a preexisting airfoil. Each one is given a weight, and each weight represents a design variable.

Cubic splines are necessary to convert the preexisting airfoil shape into a basis function used in the optimization problem. The University of Illinois at Urbana-Champaign has an excellent link to an airfoil database site. This site is http://www.aae.uiuc.edu/m-selig/ads/cood_database.html. The airfoils on this website are described by a set of data points. Each different type of airfoil varies in the number of data points given and in airfoil orientation. For this research project cubic splines are used to convert the unstandardized, varying data into a 141 data points with the same orientation. Thus, each airfoil taken from the website is transformed into a basis function sharing the same number of points and values in x coordinates.

III. Results

A. Conventional UAV Study

A study of the conventional UAV is absolutely necessary. It provides a basis of comparison for the buckle-wing UAV design. In order to make a true comparison, the same criteria have to be implemented in both studies. Then, an evaluation can be made quantitatively and graphically to demonstrate the effectiveness of the buckle-wing UAV design.

The conventional UAV problem is, quite understandably, less complicated than the buckle-wing problem. The optimization problem for the conventional UAV involves a multiobjective design of a single airfoil. The conventional UAV problem optimizes both range/endurance and maneuverability for the single wing. In contrast, the buckle wing involves multilevel, multiobjective optimization. It optimizes the fused airfoil for its range and endurance performance and the split airfoil for maneuverability. Consequently, the conventional UAV optimization problem is computationally much less expensive and generating its Pareto curve takes significantly less time. Hence, the conventional UAV problem was studied first.

The conventional UAV problem contains eight airfoil basis functions. Because the conventional wing problem is not computationally expensive compared to the buckle wing, it can include more design variables. Eight sundry airfoils were chosen for the first single-wing optimization problem shown in Figure 2. Basically, these eight were chosen because they cover a gambit of families of airfoil shapes. The only criterion for selecting them was that they are designed for small aircraft use. This meant that the airfoils had to be designed for low Reynolds number, and that they had to be thin airfoils.

Because the buckle-wing problem is computationally more expensive, only three of the eight airfoils could be used for a Pareto comparison with the buckle-wing problem. Performing the conventional study first also helped in order to discover which three airfoils should be used as basis functions for the buckle wing. Basis

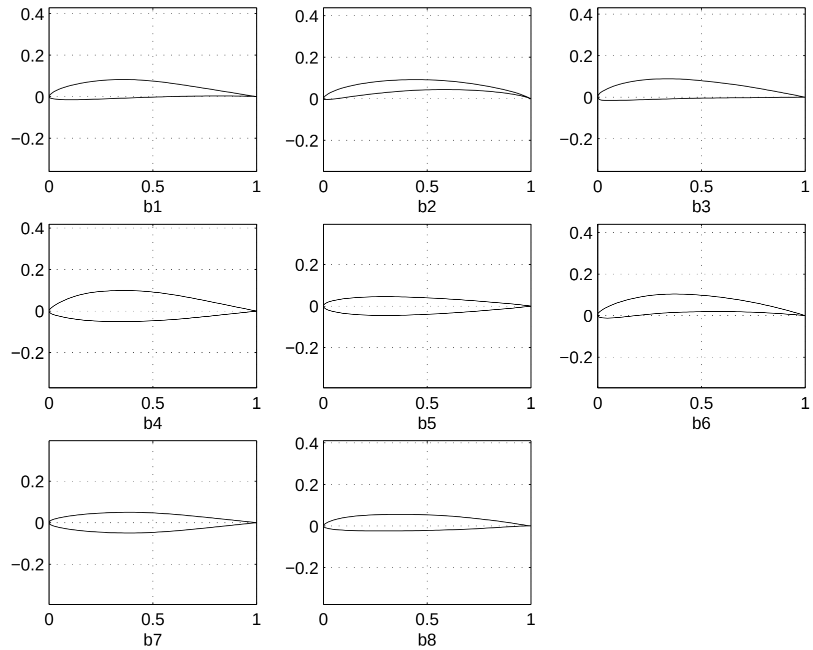

Figure 2. Basis Functions b_1, b_2, ..., b_8

function b_2 was clearly dominant. All the others played very little role in the optimization process. Still b_6 and b_8 were selected as dominant over the remaining basis functions.

Because basis function b_2 was dominant over all the other bases in the multiobjective studies, a new set of airfoil basis was required. Before choosing the new basis functions, the next step of this research was to compare conventional UAV Pareto curve to that of the buckle wing.

B. Buckle-Wing Study

The buckle-wing problem was solved and its Pareto curve established. The three basis functions for this buckle-wing problem are b_2, b_6, and b_8. The starting point x_0 = [0.5 0.5 0.5 1.0]. The utopian point f^u= [5.0 300]. The convergence tolerances = 1e-7.

Figure 3 represents the buckle-wing's optimized airfoils at the two end points of the Pareto curve. The left side of the figure shows these airfoils in the fused configuration with their cl/cd_{fused} values. The right side shows these airfoils in the split configuration with their cl_{split} values.

Figure 4 shows the conventional and buckle-wing Pareto curves for comparison. The conventional problem generated Pareto points between (142.8, 1.79) and (155.5, 1.65). The buckle-wing problem generated Pareto points between (142.8, 2.55) and (156.4, 2.31). A linear interpolating curve was used to generate the two Pareto curves. The Pareto shift is evident from the two curves. cl values varied from 1.65 to 2.31 in the conventional problem and they varied from 2.31 to 2.55 in the buckle-wing problem. Note that the buckle wing is able to obtain the same cl/cd values. The average cl difference was calculated where the two Pareto domains overlap. The average cl improvement between the two curves is 0.74 or 42.5%.

The optimization problems were not very interesting because most of the design variables (i.e., basis functions) were zeros. Two of the three airfoils comprising the problems' basis functions were disappointing. With the exception of b_2 the Eppler 61 airfoil, the basis functions were poor selections for a high cl. Therefore, research continued with an extended goal to find a more interesting design problem (i.e. nonzero design variables in the solution) and higher cl results. Further research continued to bolster the project's success.

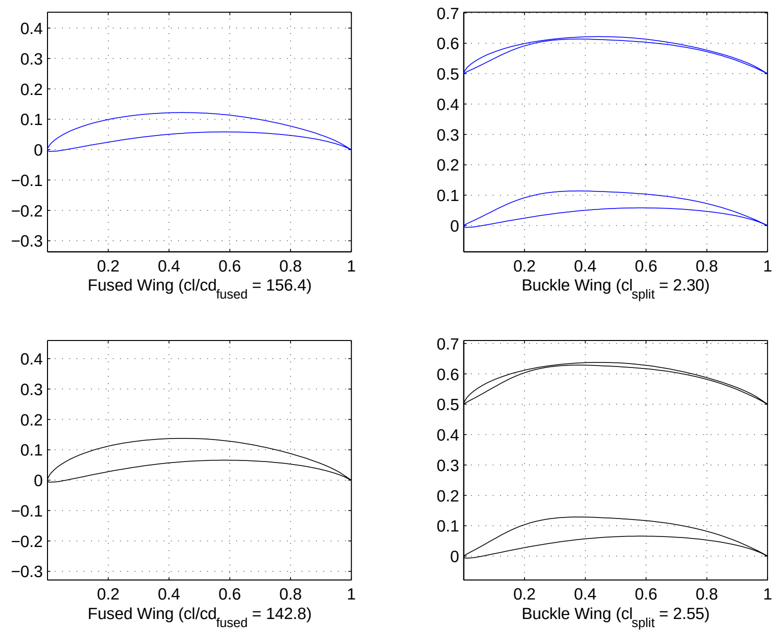

Figure 3. Optimized Buckle-Wing Shapes in Both Configurations

C. Alternate Airfoil Basis Study for Conventional UAV Design

The new goal was to find a better buckle-wing UAV design based on two new criteria. First, the design should produce higher cl and cl/cd results. Second, the optimization problem should involve all three basis functions in the optimization process. Each applicable family of airfoils from the website http://www.aae.uiuc.edu/m-selig/ads/cood_database.html was opened and analyzed visually. The family or series of airfoils, which passed the first visual check of having a thin asymmetric, cambered shape, was then scrutinized more thoroughly. The next visual checks involved comparing cambers. These visual checks were based on experience and intuition from the previous results. Finally, eight new airfoils were selected for high lift. Figure 5 shows these airfoil shapes and names. These eight comprised the alternate airfoil basis design problem.

Table 2 depicts the criteria used in selecting the most dominant three. Clearly, x_3, x_7, and x_8 were the most frequently used. The corresponding names of these three dominating airfoils are E-61, GOE804, and fxmod74. In this study one airfoil did not dominate all eight, but three played important roles in generating the Pareto curve. In fact, x_1 played a minor role as well. These three airfoils provided the basis functions for the buckle-wing problem and Pareto curve comparison.

Table 2. Nonzero Weights for the Conventional Alternate Study - 8 Bases Optimization Trials

Basis Function	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Pareto Trials (18)	5	0	15	1	1	1	13	16
Total Trials (37)	11	4	32	6	5	4	26	35

These three airfoils were among the best airfoil shapes for lift. The Eppler 61 is the best of the Eppler airfoil shapes for lift. Gottingen (EA 8) 804 proved to be the best airfoil basis function from that Gottingen

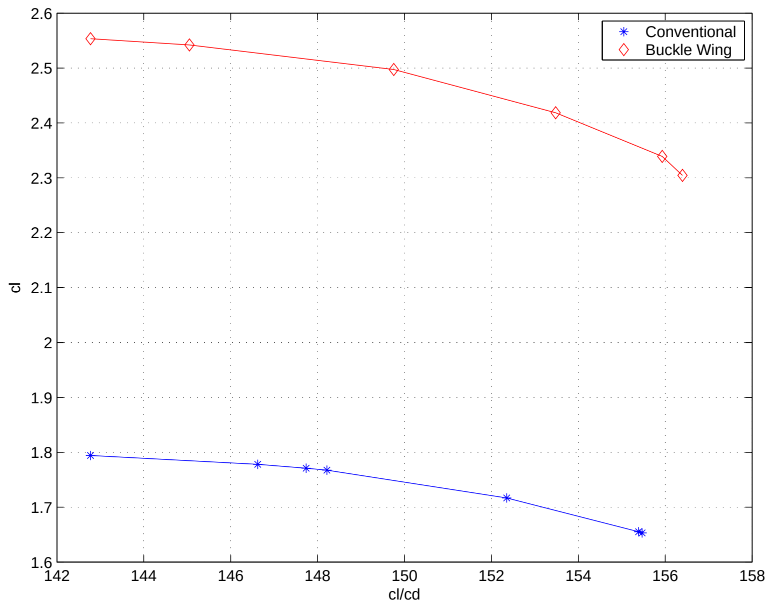

Figure 4. Pareto Curves with Bases b_2, b_6, b_8

for lift. Wortmann made modifications to its FX 74-CL5-140 high lift airfoil for even higher lift. Thus, some of the best designs were achieved.

D. Conventional UAV Study with Three Basis Functions x_3, x_7, and x_8

The Pareto curve for the conventional UAV design was generated using the three dominant bases. Tables 3 and 4 display the complete results for this problem. The Pareto curve of the this problem was established for comparison. Figure 6a illustrates the two Pareto curves for conventional design using both eight and three bases. For the most part the two curves are identical; however, the eight-bases case is able to generate better results in two areas of the curve. This was anticipated from the small contributing role that the other five basis functions played in the optimization process and basis function x_1 in particular.

Using the alternate set of airfoil bases, the Pareto curve is observed to be superior to that of the original problem shown in Figure 6b. The optimizer was able to achieve the same domain for cl/cd up to a maximum value at 155.5. Its range for cl well surpasses 1.79. It achieves cl values through 2.15. Furthermore, its cl surpasses every section where the two curves' domain overlap.

E. Buckle-Wing Problem with Alternate Bases

The buckle-wing problem using the alternate airfoil bases was solved using the knowledge learned from the previous problems. The same starting point, bounds, utopian point, and tolerances were used. The optimizer converged at local minima for nearly half of the trials. Therefore, other starting points were used to find the global minima for these problematic trials. The complete results are found in the Tables 5, 6, and 7. The Pareto curve was established and compared to the conventional UAV design.

Figure 6c shows the successful Pareto curve shift. The conventional problem generated Pareto points between (115.9, 2.15) and (155.5, 1.66). The buckle-wing generated points between (99.9, 3.37) and (154.7, 2.41). Each Pareto curve covered approximately the same cl/cd domain. The curves were constructed using a linear piecewise fit. The average cl improvement between the two curves was 0.803 or 41.7%.

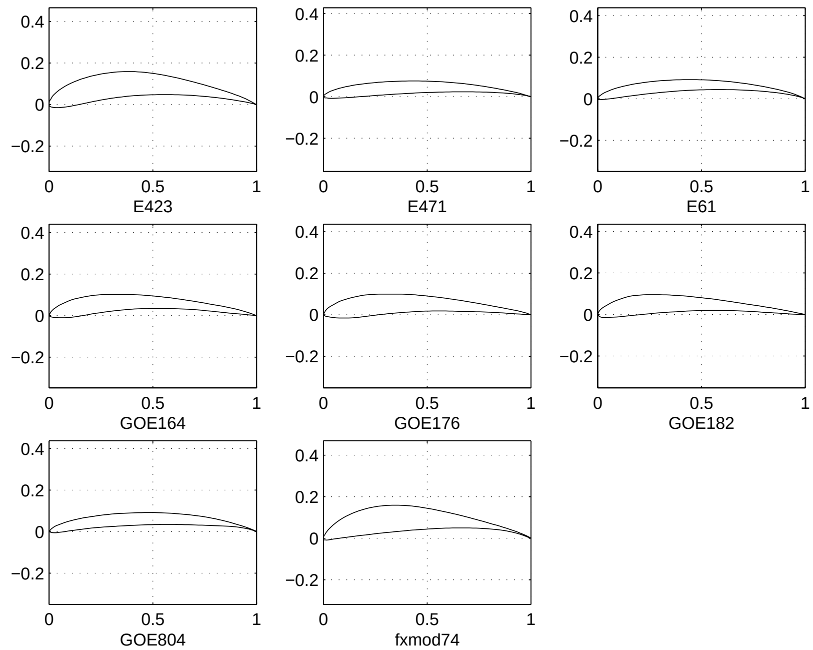

Figure 5. Eight New Basis Functions x_1, x_2,..., x_8

Compared to the first buckle-wing trial using the original bases, this buckle-wing Pareto curve was able to generate significantly larger *cl* values and a larger gamut for the curve. Figure 6d compares the two buckle-wing curves. Using alternate bases the buckle-wing Pareto curve produced higher *cl* values.

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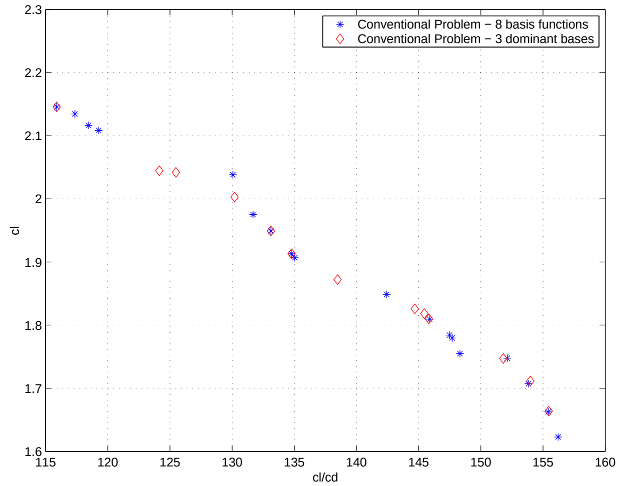
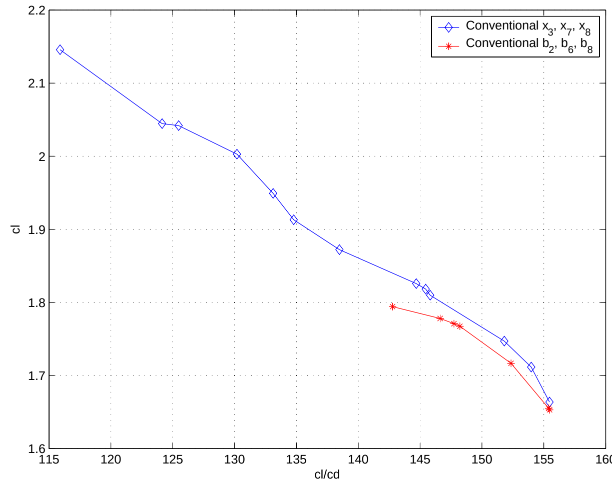
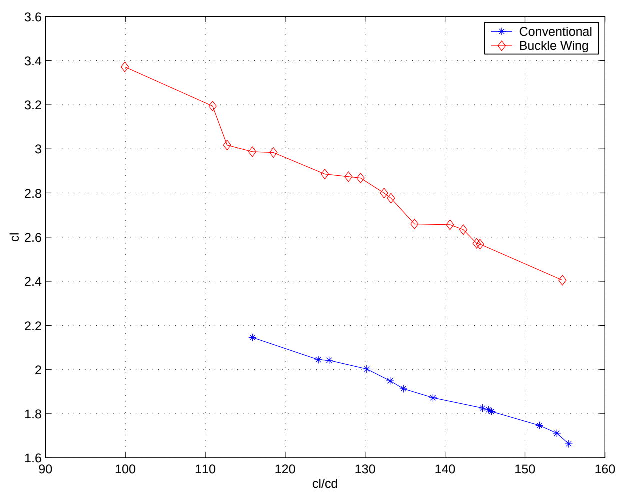


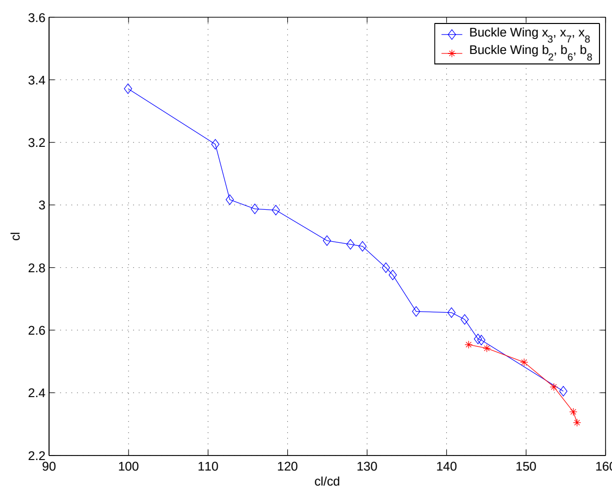
(a) Pareto Curves with High Lift Basis Functions



(b) Conventional Pareto Curves with Different Basis Functions



(c) Pareto Curves with bases E61, GOE804, fxmod74



(d) Buckle-Wing Pareto Curves

Figure 6. Pareto Curve Comparisons

Table 3. **Conventional Study - 3 Alternate Bases, Starting Point x_0=[0.5 0.5 0.5 1.0] Unless Otherwise Specified**

$\overline{f}$	cl	cl/cd	x_1	x_2	x_3	x_4	Pareto	Cons
[1.4 150]	1.4713	149.3994	0.5214	0.5303	0.1080	1.0120	no	15
[1.4 160]	1.4044	155.2235	1.1086	0.0190	0.0000	1.1194	no	3,15
[1.4 170]	1.3966	155.1372	1.0851	0.0353	0.0000	1.4954	no	3,15
[1.5 140]	1.7356	149.2438	0.9124	0.4448	0.0676	0.8459	no	15
[1.5 150]	1.4555	149.3478	0.4721	0.4721	0.1070	1.0296	no	14
x_0 = [.91 .445 .0676 1]								
[1.5 150]	1.6637	155.4560	1.0673	0.2864	0.0179	0.8909	yes	15
f^u = [4 300] and x_0 = [.91 .445 .0676 1]								
[1.5 140]	1.6746	150.9383	0.8859	0.4216	0.0610	0.9316	no	15
[1.5 150]	1.6803	150.7933	0.8743	0.4250	0.0711	0.9947	no	15
f^u = [5 400] and x_0 = [.91 .445 .0676 1]								
[1.5 140]	1.7047	150.0777	0.8837	0.4417	0.0693	0.9709	no	none
f^u = [6 500] and x_0 = [.91 .445 .0676 1]								
[1.5 140]	1.5950	147.5961	0.7501	0.4349	0.0954	0.9789	no	14,15
[1.6 140]	1.6768	143.2924	1.1583	0.0000	0.1689	0.9451	no	2,14,15
[1.6 150]	1.4982	149.2095	0.4596	0.6356	0.0957	1.0727	no	none
[1.6 160]	1.4232	154.9497	1.1452	0.0000	0.0000	1.1263	yes	2,3,14,15
[1.6 170]	1.3847	154.9758	1.0495	0.0600	0.0000	1.5008	no	none
x_0 = [1 0.4 0.1 1]								
[1.6 150]	1.7115	153.9819	1.0907	0.2708	0.0046	0.9204	yes	14,15
[1.7 140]	1.8182	145.4517	1.3446	0.0000	0.1315	0.9091	yes	2,15
[1.7 150]	1.4724	141.3505	0.4096	0.6098	0.1335	1.1751	no	14
[1.7 160]	1.4161	153.4753	0.7437	0.3168	0.0608	1.2184	no	none
x_0 = [1.1334 .2730 .0379 1]								
[1.7 140]	1.8090	145.0327	1.1885	0.2583	0.0520	0.9161	no	14,15,16
[1.7 150]	1.7470	151.8031	1.1334	0.2730	0.0379	1.0000	yes	none

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Table 4. Conventional Study - 3 Alternate Bases, *continued*

$\overline{f}$	cl	cl/cd	x_1	x_2	x_3	x_4	Pareto	Cons
x_0 = [1.4775 0 0 1]								
[1.7 140]	1.8182	145.4517	1.3446	0.0000	0.1315	0.9091	(used)	2,15
[1.7 150]	1.7440	151.6906	1.1809	0.2585	0.0113	0.9662	no	14,15
[1.9 130]	1.9491	133.1222	0.0000	0.9349	0.5344	0.9554	yes	1,14,15
[1.9 140]	1.8462	137.0174	0.2806	0.7243	0.4082	1.0497	no	15
[1.9 150]	1.8098	145.8147	1.3364	0.0000	0.1316	1.0837	yes	none
x_0 = [1.3364 0 .1316 1]								
[1.9 150]	1.8257	144.6893	1.3483	0.0000	0.1341	1.0354	yes	2,14,15
[1.9 140]	1.8722	138.4813	1.2897	0.0000	0.2103	1.0253	yes	2,14,15,16
[2.0 120]	2.0446	124.1472	0.2480	0.5618	0.6902	0.9554	yes	14,16
[2.0 130]	2.0028	130.1928	0.0004	0.9083	0.5913	0.9972	yes	14,15,16
[2.0 140]	1.9130	134.7814	0.0000	0.9346	0.5091	1.0870	yes	14,15
[2.1 110]	2.1457	115.8915	0.5613	0.0000	0.9387	0.9492	yes	2,10,14,16
[2.1 120]	2.0995	115.0208	0.1737	0.4767	0.8451	1.0006	no	NR
[2.1 130]	2.0419	125.4845	0.2267	0.5891	0.6842	1.0645	yes	NR
[2.3 110]	2.1457	115.8915	0.5613	0.0000	0.9387	1.2204	x	2,10,14,16
[2.3 120]	2.1457	115.8915	0.5613	0.0000	0.9387	1.2204	x	2,10,14,16
[2.3 130]	2.1457	115.8915	0.5613	0.0000	0.9387	1.2204	x	2,10,14,16

$F = x_4 - \alpha(w_1 f_1 + w_2 f_2)$, $\quad w_i = 1/(f_i^u - \overline{f}_i)$

f^u = [3.0 200] Unless Otherwise Specified

Convergence Tolerances = 1e-7; Active Constraints (Cons)

Constraints 1-4: Lower Bounds = $[\bar{0}]$

Constraints 5-8: Upper Bounds = [1.5 1.5 1.5 3.0]

Constraint 9: $0 \geq cm$,

Constraint 10: $0.2 \geq max(y_{new})$, where y_{new} is the airfoil's y coordinate,

Constraint 11: $0.2 \geq -min(y_{new})$,

Constraint 12: $0.3 \geq max(y_{new}) - min(y_{new})$,

Constraint 13: $-0.1 \geq -max(y_{new}) + min(y_{new})$,

Constraint 14: $x_4 \geq w_1(f_1^u - f_1)$

Constraint 15: $x_4 \geq w_2(f_2^u - f_2)$

Constraint 16: $1.5 \geq x_1 + x_2 + x_3$

Table 5. **Buckle-Wing Alternate Bases, Starting Point x_0=[0.5 0.5 0.5 1.0]**

$\overline{f}$	cl	cl/cd	x_1	x_2	x_3	x_4	Pareto	Cons
[2.0 140]	2.1687	149.2055	0.5803	0.5227	0.1024	0.9438	no	NR
[2.0 150]	2.0913	153.4535	0.8615	0.3186	0.0054	0.9770	no	NR
[2.0 160]	1.9530	154.8810	0.9137	0.1788	0.0000	1.0366	yes	NR
[2.0 170]	1.9418	154.9397	0.8572	0.2285	0.0000	1.1185	yes	3,11
[2.1 130]	2.3440	145.4206	0.6564	0.5968	0.0804	0.9159	no	NR
[2.1 140]	2.2026	148.9075	0.5660	0.5689	0.0963	0.9646	no	NR
[2.1 150]	2.1591	153.0544	0.9026	0.2789	0.0378	0.9796	yes	NR
[2.2 150]	2.1837	149.0955	0.6111	0.5021	0.1022	1.0060	no	NR
[2.2 160]	2.0894	149.3864	0.5841	0.4561	0.1086	1.0758	no	NR
[2.2 170]	1.9419	154.9395	0.8573	0.2283	0.0000	1.1159	yes	11
[2.5 140]	2.4421	136.2838	0.3994	0.5490	0.3518	1.0232	no	NR
[2.5 150]	2.4798	148.7860	0.8483	0.5087	0.0718	1.0081	yes	NR
[2.5 160]	2.2790	147.6265	0.5855	0.6172	0.0852	1.0884	no	NR
[2.5 170]	2.1109	148.6320	0.3087	0.7843	0.0836	1.1644	no	11
[2.6 130]	2.6790	135.5930	0.4102	0.6195	0.4118	0.9671	no	NR
[2.6 140]	2.6341	142.2728	1.3279	0.0000	0.1663	0.9858	yes	2,10,11
[2.6 150]	2.3936	137.1001	0.3947	0.5819	0.3074	1.0860	no	10,11
[2.7 130]	2.7423	133.3630	0.0838	0.9544	0.4407	0.9816	yes	NR
[2.7 140]	2.6510	136.5926	0.2280	0.7859	0.4109	1.0213	no	NR
[2.7 150]	2.5719	143.9487	0.8267	0.6176	0.0556	1.0557	yes	NR
[2.7 160]	2.3395	138.0592	0.4058	0.5463	0.2981	1.1567	no	10,11
[2.9 120]	2.8932	124.0234	0.4272	0.4280	0.6449	1.0034	no	11
[2.9 130]	2.8742	127.9077	0.1349	0.6889	0.6604	1.0123	yes	11
[2.9 140]	2.7997	132.3587	0.0000	0.9256	0.5512	1.0478	yes	1,10,11
[3.0 120]	2.9836	118.5199	0.7035	0.0000	0.7965	1.0082	yes	NR
[3.0 130]	2.8861	124.9580	0.4291	0.4387	0.6323	1.0571	yes	NR
[3.0 140]	2.8678	129.4216	0.0296	0.8576	0.6128	1.0661	yes	NR
[3.0 150]	2.7764	133.2331	0.0000	0.9304	0.5360	1.1118	yes	NR
[3.0 160]	2.6596	136.1690	0.4624	0.6031	0.3758	1.1702	yes	10,11
[3.2 110]	3.1940	110.9140	0.3206	0.0000	1.1794	1.0034	yes	NR
[3.2 120]	3.0731	107.3064	0.0000	0.0000	1.3497	1.0705	no	NR
[3.2 130]	3.0171	112.7285	0.2966	0.0000	1.1077	1.1016	yes	11
[3.2 140]	3.1940	110.9140	0.3206	0.0000	1.1794	1.0034	no	NR
[3.2 150]	3.0731	107.3064	0.0000	0.0000	1.3497	1.0705	no	NR
[3.2 160]	3.0171	112.7285	0.2966	0.0000	1.1077	1.1016	(used)	NR
[3.2 170]	2.9874	115.8806	0.2397	0.4421	0.8182	1.1507	yes	11

continued next page

Table 6. **Buckle-Wing Alternate Bases,** ***continued***

$\overline{f}$	cl	cl/cd	x_1	x_2	x_3	x_4	Pareto	Cons
[3.4 100]	3.3714	99.9400	0.0000	0.0000	1.5000	1.0179	yes	2,7,10,11
[3.4 110]	3.3129	99.6543	0.0000	0.0000	1.4704	1.0545	no	NR
[3.4 120]	3.2531	103.4741	0.0020	0.0000	1.4387	1.0918	no	NR
[3.6 90]	3.3714	99.9400	0.0000	0.0000	1.5000	1.1633	(used)	2,7,10,11
[3.6 100]	3.3714	99.9400	0.0000	0.0000	1.5000	1.1633	(used)	2,7,10,11
[3.6 110]	3.3714	99.9400	0.0000	0.0000	1.5000	1.1633	(used)	2,7,10,11
[3.6 120]	3.3714	99.9400	0.0000	0.0000	1.5000	1.1633	(used)	2,7,10,11
[3.6 130]	3.3300	97.0000	NR	NR	NR	NR	no	NR

continued below

Table 7. Buckle-Wing Alternate Bases, Starting Point x_0=[1.0 0.3 0.1 1.0]

$\overline{f}$	cl	cl/cd	x_1	x_2	x_3	x_4	Pareto	Cons
[2.0 140]	2.2236	151.9241	0.9271	0.3062	0.0322	0.9255	no	10,11
[2.0 150]	2.0873	154.3659	1.1809	0.0000	0.0000	0.9709	no	2,3,10,11
[2.0 160]	1.9941	155.1090	1.0784	0.0399	0.0000	1.0349	no	3,11
[2.2 150]	2.2258	151.3820	1.2672	0.0077	0.0000	0.9908	yes	3,10,11
[2.5 130]	2.6561	140.6166	1.3081	0.0000	0.1919	0.9375	yes	NR
[2.5 140]	2.5682	144.3618	0.8637	0.5731	0.0588	0.9727	yes	10,11
[2.5 150]	2.4795	148.7727	1.0580	0.3287	0.0484	1.0082	no	NR
[2.5 160]	2.4049	154.6748	1.0770	0.2905	0.0245	1.0380	yes	10

Convergence Tolerances = 1e-7

f^u = [5.0 300] $w_i = 1/(f_i^u - \overline{f}_i)$

Constraints 1-4: Lower Bounds = $[\bar{0}]$

Constraints 5-8: Upper Bounds = [1.5 1.5 1.5 3.0]

Constraint 9 : $0.9 \geq cl_{fused}$

Constraint 10: $x_4 \geq w_1(f_1^u - f_1)$

Constraint 11: $x_4 \geq w_2(f_2^u - f_2)$

Constraint 12: $1.5 \geq x_1 + x_2 + x_3$